Analyzing Multiagent Interactions in Traffic Scenes via Topological Braids

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Abstract—We focus on the problem of analyzing multiagent interactions in traffic domains. Understanding the space of behavior of real-world traffic may offer significant advantages for algorithmic design, data-driven methodologies, and benchmarking. However, the high dimensionality of the space and the stochasticity of human behavior may hinder the identification of important interaction patterns. Our key insight is that traffic environments feature significant geometric and temporal structure, leading to highly organized collective behaviors, often drawn from a small set of dominant modes. In this work, we propose a representation based on the formalism of topological braids that can summarize arbitrarily complex multiagent behavior into a compact object of dual geometric and symbolic nature, capturing critical events of interaction. This representation allows us to formally enumerate the space of outcomes in a traffic scene and characterize their complexity. We illustrate the value of the proposed representation in summarizing critical aspects of real-world traffic behavior through a case study on recent driving datasets. We show that despite the density of real-world traffic, observed behavior tends to follow highly organized patterns of low interaction. Our framework may be a valuable tool for evaluating the richness of driving datasets, but also for synthetically designing balanced training datasets or benchmarks.

I. INTRODUCTION

Driven by the widespread interest in autonomous driving [2], the robotics community has paid special attention to the problem of navigation in traffic environments [34, 11, 15, 35]. These environments pose unique challenges due to their high dimensionality and the complexity of modeling human behavior whereas the safety-critical nature of the domain sets high standards for performance and validation.

Despite these complications, real-world traffic scenes often feature significant structure. Vehicles follow designated lanes and traffic is regulated by traffic signals and signs or coordinated via turn indicators. Driver behavior can often be modeled as rational, characterized by risk aversion and efficiency-seeking objectives. Recent work has leveraged these observations in the design of data-driven behavior prediction and planning frameworks [42, 7, 34, 17]. To perform robustly in the real world, such frameworks require large, balanced datasets containing highly diverse behaviors.

Further, to measure their performance, it is important to understand the diversity of behavior that is expected in the real world. However, to accomplish the goal of diversity, it is important to understand the space of behavior.

Our key insight is that due to their behavioral and geometric structure, multiagent behavior in traffic scenes exhibits topological properties: common events like overtaking, merging, crossing an intersection (Fig. 3) constitute multiagent interactions that generate topological signatures [3]. In this work, we abstract multiagent behavior into a topological braid [5], a compact and interpretable topological object with symbolic and geometric descriptions. Building on past work on the use of braids for multiagent navigation [26], we make the following contributions. First, we adapt the representation of Mavrogiannis and Knepper [26] to structured domains like driving environments through a new rigorous mathematical presentation. We then study its computational properties, and discuss how a measure of complexity based on braids [13] may capture the interactivity of a traffic scene. We show that our framework can be applicable to complex scenes through a case study on real-world intersections and roundabouts [6, 20]. We cluster the behaviors exhibited in these scenes into braids, and characterize their complexity. We find that in the majority of scenes, a few simple braids dominate, indicating a low degree of interaction despite the high traffic density. Our methodology can be valuable for the analysis and design of road networks, the design and benchmarking of data-driven frameworks for prediction and planning, the evaluation and generation of driving datasets.

Fig. 1: This work proposes a formal framework for the characterization of multiagent behavior in driving domains. Complex multiagent interactions encountered in real-world driving domains such as a roundabout can be compactly represented as topological braids (right).

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II. RELATED WORK

Recent work on behavior prediction and decision making for autonomous driving applications has leveraged discrete, semantic representations of traffic behavior. For instance, Wang et al. [44] classify discrete driving styles using a variant of hidden Markov models (HMM). Gadepally et al. [15] also use HMM to estimate long-term driver behaviors from a sequence of discrete decisions. Others, such as Konidaris et al. [19] and Shalev-Shwartz et al. [37], propose using learned symbolic representations for high-level planning and collision avoidance, via a hierarchical options model. Similarly, Shu et al. [38] learn a latent representation of interactions. While these works uncover discrete representations of driving behavior, they either require large datasets to learn discrete modes or specify them manually without harnessing the rich geometric structure of the environment.

Another body of work has focused on tools for testing and validation in realistic settings, leveraging a semantic-level understanding of interactions. Tian et al. [42] model traffic at unsignaled intersections using tools from game theory and propose a verification testbed for navigation algorithms. Liebenwein et al. [22] propose a framework for safety verification of driving controllers based on compositional and contract-based principles. Hsu et al. [17] investigate how velocity signals generated by Markov decision processes affect interaction dynamics at intersections. DeCastro et al. [11] construct a representation of multi-vehicle interaction outcomes based on latent parameters using a generative model. Tolstaya et al. [43] propose an Interactivity score that enables the identification of interesting interactive scenarios for training generative models. Our work is similar in spirit and complementary to this latter line of work. We also approach a notion of interactivity between agents. However, instead of investigating statistical properties of distributions, we focus on the aspect of the representation, through the introduction of tools from low-dimensional topology.

Recently, roboticists have been increasingly making use of topological representations to model the rich structure of real-world domains. These include knotting [23], untangling [16] or knot planning [45], aircraft conflict resolution [18] or multiagent navigation [12]. Some works leverage insights from homotopy theory [9, 4], persistent homology [32] and fiber bundles [30]. Some other works make use of topological abstractions such as invariants [28, 24, 34] and braids [27, 26] as representatives of multiagent motion primitives for prediction and planning. In this paper, we are following up on this latter body of work work by employing topological braids as an abstraction of traffic behavior. While previous work considered simplified simulation domains [27, 26], in this paper, we adapt the braid presentation to accommodate rich traffic environments such as real-world intersections or roundabouts. To the best of our knowledge this paper is the first to investigate the applicability of braids as primitives for multiagent behavior in realistic real-world environments. Note that our work builds upon insights from earlier work [25] targeting the planning domain.

![Braid Group Example](image)

(a) Generators of $B_n$.  
(b) Example of composition.

Fig. 2: Presentation of the braid group, $B_n$.

III. ABSTRACTING DRIVING INTERACTIONS AS TOPOLOGICAL BRAIDS

We introduce a representation based on topological braids [1], that captures critical interaction events in street environments (e.g., overtaking, merging, crossing). This representation describes such interactions as sequences of symbols describing topological relationships between agents; any possible interaction manifests as a unique symbolic representation of their trajectories. Our representation adapts the presentation of Mavrogiannis and Knepper [26] to real-world traffic domains through theoretical developments.

A. Domain

Consider a structured domain $\mathcal{Q} \subseteq \mathbb{R}^2$ where $n > 1$ agents are navigating from time $t = 0$ to time $t_\infty$. Define by $g_i \in \mathcal{Q}$ the position of agent $i \in \mathcal{N} = \{1, \ldots, n\}$ with respect to a fixed reference frame. Agent $i$ follows a trajectory $\xi_i : [0, t_\infty] \rightarrow \mathcal{Q}$. Collectively, agents follow a system trajectory $\Xi = (\xi_1, \ldots, \xi_n)$. This trajectory is a detailed representation of the collective strategy that agents followed to avoid each other while following their paths. Their strategy can be summarized as a set of discrete relationships, such as the passing sides or crossing order of agents. These relationships are formed as a result of the geometric structure of the environment, traffic regulations, and agents’ policies. In this paper, we show that such relationships feature topological properties that can be succinctly captured by the representation of topological braids [1, 5].

B. Topological Braids

A braid is a tuple $b_f = (f_1, \ldots, f_n)$ of functions $f_i : I \rightarrow \mathbb{R}^2 \times I$, $i \in \mathcal{N}$, defined on a domain $I = [0, 1]$ and embedded in a euclidean space $(\hat{x}, \hat{y}, \hat{t})$. These functions, called *strands*, are monotonically increasing along the $\hat{t}$ direction, satisfying the properties: (a) $f_i(0) = (i, 0, 0)$, and $f_i(1) = (p_f(i), 0, 1)$, where $p_f : \mathcal{N} \rightarrow \mathcal{N}$ is a permutation in the symmetric group $\mathcal{N}_n$; (b) $f_i(t) \neq f_j(t)$ $\forall$ $t \in I$, $j \neq i \in \mathcal{N}$. Two braids, $b_f = (f_1, \ldots, f_n)$, $b_g = (g_1, \ldots, g_n)$, can be composed through a composition operation (Fig. 2b): their composition, $b_h = b_f \cdot b_g$, is also a braid $b_h = (h_1, \ldots, h_n)$, comprising a set of $n$ curves, defined as:

$$h_i(t) = \begin{cases} f_i(2t), & t \in [0, \frac{1}{2}] \\ g_j(2t - 1), & t \in [\frac{1}{2}, 1] \end{cases},$$

where $j = p_f(i)$. The set of all braids on $n$ strands, along with the composition operation form a group, $B_n$, called the...
Braid group on \( n \) strands. Following Artin’s presentation [1], the braid group \( B_n \) can be generated from \( n - 1 \) primitive braids \( \sigma_1, \ldots, \sigma_{n-1} \) (see Fig. 2a), called generators, and their inverses, via composition.

A generator is a braid \( \sigma_i = (\sigma_1, \ldots, \sigma_n), \ i \in \mathbb{N} \setminus n \) for which: (a) \( \sigma_i(0) = (1, 0, 0) \), and \( \sigma_i(1) = (p_i(i), 0, 1) \), where \( p_i : \mathcal{N} \rightarrow \mathcal{N} \) is an adjacent transposition swapping \( i \) and \( i + 1 \); (b) there exists a unique \( t_c \in [0, 1] \) such that \((\sigma_i(t_c) - \sigma_{i+1}(t_c)) \cdot \hat{x} = 0 \) and \((\sigma_i(t_c) - \sigma_{i+1}(t_c)) \cdot \hat{y} > 0 \).

The inverse of \( \sigma_i \) is the braid \( \sigma_i^{-1} = (\sigma_1^{-1}, \ldots, \sigma_n^{-1}) \), \( i \in \mathbb{N} \setminus n \), for which: (a) \( \sigma_i^{-1}(0) = (1, 0, 0) \), and \( \sigma_i^{-1}(1) = (p_i(i), 0, 1) \); (b) there exists a unique \( t_c \in [0, 1] \) such that \((\sigma_i^{-1}(t_c) - \sigma_{i+1}^{-1}(t_c)) \cdot \hat{x} = 0 \) and \((\sigma_i^{-1}(t_c) - \sigma_{i+1}^{-1}(t_c)) \cdot \hat{y} < 0 \).

The identity braid \( \sigma_0 = (\sigma_1^0, \ldots, \sigma_n^0) \) is defined via a trivial permutation implementing no swap \( p_0(i) = i \), yielding \( \sigma_0^0(0) = (i, 0, 0) = (i, 0, 1) \) and it holds that \( \hat{x}t_c \in [0, 1] \) such that \((\sigma_i^0(t_c) - \sigma_{i+1}^0(t_c)) \cdot \hat{x} = 0 \) for any \( i \).

Any braid can be written as a word, i.e., a product of generators and their inverses (Fig. 2b), satisfying the relations:

\[
\sigma_i \sigma_j = \sigma_j \sigma_i, \ |j - i| > 1, \\
\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}, \ \forall \ i.
\] (2)

C. Transforming Traffic Trajectories into Braids

We will convert a system trajectory \( \Xi \) into a Cartesian object with the structure of a topological braid through a sequence of operations that retain the topological relationships among agents’ trajectories.

We define by \( \xi_i^x : [0, t_\infty] \rightarrow \mathbb{R} \) and \( \xi_i^y : [0, t_\infty] \rightarrow \mathbb{R} \) the \( x \) and \( y \) projections of \( \xi_i \). For \( t = 0 \), ranking agents in order of increasing \( \xi_i^x(0) \), \( i \in \mathcal{N} \) value defines a starting permutation \( p_s : \mathcal{N} \rightarrow \mathcal{N} \), where \( p_s(i) \) denotes the order of agent \( i \). For \( t = t_\infty \), ranking agents in order of increasing \( \xi_i^x(t_\infty) \) value defines a final permutation \( p_d : \mathcal{N} \rightarrow \mathcal{N} \), where \( p_d(p_s(i)) \) denotes the final ranking of agent \( i \). Thus the execution in \( \Xi \) can be abstracted into a transition from \( p_s \) to \( p_d \).

We denote by \( \tau : I \rightarrow [0, t_\infty] \) a time normalization function, uniformly mapping \( I = [0, 1] \) to the execution time in the range \([0, t_\infty] \). We then define the trajectory bounds as \( x_{\min} = \min_{i,t} \xi_i^x(t), \ x_{\max} = \max_{i,t} \xi_i^x(t), \) and \( y_{\min} = \min_{i,t} \xi_i^y(t) \) and \( y_{\max} = \max_{i,t} \xi_i^y(t) \).

Finally, we define a set of functions \( f_1, \ldots, f_n \), with \( f_j : I \rightarrow [0, 1] \), \( j = 1, \ldots, n \), such that:

\[
f_j(a) = \begin{cases} 
(j, 0, 0), & a = 0 \\
(f_j^x(a), f_j^y(a), a), & a \in (0, 1), \\
(p_d(j), 0, 1), & a = 1
\end{cases}
\] (4)

where

\[
\begin{align*}
\tau_1(t) &= 1 + r_1^x(t) - x_{\min} - x_{\max}, \\
\tau_2(t) &= 1 + r_2^y(t) - y_{\min} - y_{\max}
\end{align*}
\] (5)

D. Braids as Modes of Traffic Coordination

The transformation of Sec. III-C enables summarization of a traffic episode into a braid capturing multiagent collision-avoidance relationships. This braid can be written as a word, similarly to how Thiffeault [40] converted particle motion in a fluid to a braid (Fig. 3): a) we label any trajectory crossings that emerge within the \( x-t \) projection as braid generators by identifying under or over crossings (Fig. 3b); b) we arrange these generators in temporal order into a braid word. Note that alternative reference frames can be employed; we selected the \( \hat{x} \cdot \hat{t} \) plane projection for convenience.

In Fig. 3, four agents cross an intersection. The braid \( \sigma_3 \sigma_1 \sigma_2^{-1} \sigma_3^{-1} \sigma_1^{-1} \in B_4 \) is a description of how agents coordinated to avoid each other. The group \( B_4 \) contains all ways in which these four agents could possibly avoid each other. In a scene with \( n \) agents, a braid represents a mode of coordination from the set of possible modes in \( B_n \).

E. Computational Properties of the Representation

To highlight the possible computational benefits arising from the summarization of traffic episodes into braids, we study the runtime of enumerating modes of coordination as topological braids in comparison to enumerating Cartesian

Fig. 4: Transitioning from a real-world episode to a braid. The trajectories of (a) are first projected on the plane \( x-t \) (b) and then the braid \( \sigma_2 \sigma_1 \sigma_3 \sigma_2 \) is reconstructed (c).
trajectories. Consider a traffic episode of $H$ timesteps, involving $n$ agents. Each agent has $T$ options of routes to follow and $U$ actions to take at every timestep. We assume that there is at most one agent per lane, i.e., $n \leq T$. The horizon of the execution is long, and thus $n << H$. Further, $U$ is a realistically rich space of controls, and thus $n << U$, $H << U$ and $T << U$. Finally, agents are goal-driven and thus they will cross paths with each other at most once.

The number of possible Cartesian trajectories in this domain is $N_c = |T|^n(U^n)^H$. Enumerating these trajectories runs in time $O(2^n H \log U)$. For the same scene, the number of possible braids generally depends on the structure of the road network. However, we can bound the number of possible outcomes as $N_b \leq 3^2$, where the exponent is the binomial coefficient representing the number of all pairs of agents and the base represents the 3 types of possible interactions per pair that could be represented by a braid, i.e., “over-crossing”, “under-crossing”, or no crossing. This enumeration runs in time $O(2^{n^2})$.

**Theorem III.1.** The runtime of enumerating braids is lower than the runtime of enumerating Cartesian trajectories for the class of driving problems considered above.

**Proof.** We want to show that $2^{n^2} < 2^n H \log U$. This inequality is equivalent to $n < H \log U$. We assumed that $n << H$, $n << U$, therefore it should also hold that $n << H \log U$. Thus the initial inequality holds and supports the statement that the runtime of enumerating braids is significantly lower than the runtime of enumerating Cartesian trajectories.

F. Complexity of Braid Entanglement

The entanglement of the trajectories described by a braid is indicative of the complexity of the interaction between agents. We quantify braid complexity using the Topological Complexity index (TC) of Dynnikov and Wiest [13] for which we provide an informal definition below.

Assume that a braid $\beta \in B_n$ represents the collective motion of $n$ agents from initial locations $\beta(0)$ to final locations $\beta(1)$. Denote by $D^2$ a closed disk surrounding agents’ initial positions, $\beta(0)$. Define by $E$ a set of $n - 1$ disjoint arcs, anchored on the disk, and separating the agents for $t = 0$, defining $n - 1$ distinct regions in the disk (see Fig. 5). Assume that these regions are rigidly attached on the agents. As the agents follow the motion described by $\beta$ from $t = 0$ to $t = 1$, the regions dynamically deform. The image $D = \beta \cdot E$ representing the shape of the regions obtained upon applying the motion described by $\beta$ on $E$ is called a curve diagram. The norm of curve diagram $D$ is defined as the number of intersections of $D$ with the $x$ axis. Based on the above definitions, we can define the TC index of a braid $\beta \in B_n$ as:

$$TC(\beta) = \log_2(||\beta \cdot E||) - \log_2(||E||).$$

This expression is equivalent to the logarithm of the gain of intersections with the $x$-axis, upon application of a braid.

Fig. 5 depicts curve diagrams acquired upon inducing motion of two different braids on the canonical curve diagram $E$.

IV. A CASE STUDY ON TRAFFIC DATASETS

We demonstrate how braids may abstract traffic episodes through a case study on real-world datasets.

A. Datasets

We consider the inD [6] and roundD [20] datasets. These datasets contain trajectories of vehicles, pedestrians and bicycles from traffic scenes of the German road network. The inD dataset contains trajectories from four intersection scenes whereas the roundD dataset contains trajectories collected from four roundabout scenes. Both datasets were extracted from drone footage in 25fps, through the use of computer vision techniques, yielding an estimated positional error in the order of 10cm. A top view of the 8 scenes is shown in Fig. 6. The dimensions of the scenes are shown in Table I.

B. Methodology

We split each scene into a set of sequential episodes, sweeping the whole duration of the recording. Each episode has a fixed duration of $\Delta T = 10s$, containing trajectories of simultaneously navigating agents. We filtered out stationary agents (from qualitative inspection, we assumed that agents moving with a speed lower than $14\text{m/s}$ are stationary) and agents that are too far from each other (agents that kept a minimum distance greater than $d_{min} = 10\text{m}$ throughout the episode). Details about the episodes considered are shown in Table I. Using the framework of Sec. III-B, we then abstracted
Fig. 7: Cumulative density of TC (Topological Complexity index) in intersections (a) and roundabouts (b).

(a) inD scenes.  
(b) roundD scenes.

Fig. 8: Frequency of unique braids in intersections (a) and roundabouts (b), arranged in order of increasing TC (Topological Complexity index).

(a) inD scenes.  
(b) roundD scenes.

the trajectory of each episode into a braid. We shortened the extracted braids by leveraging the braid relations of eq. (2). Finally, we computed the TC score for each braid. We performed all computations using the Braidlab package [41].

C. Analysis

Table I lists the number of unique braids, and the statistics of TC per scene. We see that the set of episodes in each scene is clustered into a small number of unique braids, describing vehicles’ interaction patterns. This highlights that real-world traffic tends to collapse to a small set of outcomes. The extracted braids are mapped onto the TC values on the right. Fig. 9 depicts episodes of varying TC, drawn from the two datasets, along with their braid representatives and TC scores. We qualitatively see how complex interactions get mapped onto higher TC values.

Fig. 7 shows the empirical cumulative density of TC across the inD and roundD dataset scenes. We generally see that each scene has a distinct complexity growth pattern but in both datasets, about 60% of episodes are concentrated below $TC = 1.5$. This is highlighted in Fig. 8 which shows the relative frequency of unique braids per scene, organized in order of increasing TC. We see that the mass of the frequency is concentrated on the left side for both plots, suggesting that the majority of episodes feature a relatively low degree of interaction. This indicates that despite the dense traffic exhibited in the datasets (table I), the vast majority of episodes involve traffic that is orderly and well organized. This is an artifact of the underlying spatiotemporal structure (geometry, traffic rules, driving styles).

D. Discussion

Our representation enables enumeration of the types of multiagent interactions that are theoretically possible in traffic domains in a compact and interpretable form. Given traffic data, it allows us to extract the subset of those interactions that are empirically likely. This can inform the design of algorithmic design, benchmarking and even road networks.

Our framework can be valuable for characterizing a traffic dataset as it allows us to determine how much support a dataset provides over the space of theoretically possible behavior in a domain. Understanding this support may help debugging data-driven approaches (for e.g., prediction and planning) but also guide the process of synthetically generating simulated scenarios to produce diverse datasets.

Our framework is complementary to alternative approaches for characterizing interaction, such as the interactivity score [43] (mutual information) and distribution-based KL-divergence. The Interactivity score may miss crucial interaction events: scores can be large when there is high correlation between two trajectories (e.g., one car following another), but small when trajectories are dissimilar (e.g., cars crossing an intersection). In contrast, TC will account for these situations through the consideration of the underlying topological structure. Further, our framework may be directly applicable to any traffic dataset [14, 8, 10] without additional modifications. Thus, it may complement temporal logic approaches for trajectory labeling [33, 21] which often require involved and domain-specific mathematical treatment [36].

V. Conclusions

We presented a topological framework for the characterization of multiagent interactions in traffic scenes. To illustrate its value, we presented a case study demonstrating the types of behaviors that can be observed in two real-world traffic datasets. While we applied our framework to traffic scenes, it may be useful to other multiagent domains such as pedestrian tracking [31] or sports analysis [39].

Since our goal was to provide a proof-of-concept demonstration, specific parameters such as the projection plane for braids, the episode duration, the maximum-distance threshold between agents and the minimum moving distance threshold were empirically selected. These parameters could be further optimized or adapted to reflect the context of a particular scene (e.g., speed limits).

<table>
<thead>
<tr>
<th>Scene</th>
<th>Dimensions ($m^2$)</th>
<th>Episodes</th>
<th>Agents/Episode (M, SD)</th>
<th>Unique braids</th>
<th>TC (M, SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inD 1</td>
<td>131 × 130</td>
<td>347</td>
<td>3.62 ± 1.76</td>
<td>155</td>
<td>1.82 ± 0.58</td>
</tr>
<tr>
<td>inD 2</td>
<td>59 × 64</td>
<td>254</td>
<td>2.92 ± 1.00</td>
<td>62</td>
<td>1.48 ± 0.55</td>
</tr>
<tr>
<td>inD 3</td>
<td>65 × 45</td>
<td>366</td>
<td>2.62 ± 0.90</td>
<td>41</td>
<td>1.28 ± 0.66</td>
</tr>
<tr>
<td>inD 4</td>
<td>79 × 67</td>
<td>174</td>
<td>4.10 ± 1.51</td>
<td>99</td>
<td>1.79 ± 0.28</td>
</tr>
<tr>
<td>round 1</td>
<td>99 × 143</td>
<td>58</td>
<td>3.16 ± 1.45</td>
<td>30</td>
<td>1.20 ± 0.84</td>
</tr>
<tr>
<td>round 2</td>
<td>99 × 122</td>
<td>59</td>
<td>3.85 ± 1.75</td>
<td>32</td>
<td>1.54 ± 0.50</td>
</tr>
<tr>
<td>round 3</td>
<td>127 × 69</td>
<td>574</td>
<td>4.36 ± 2.28</td>
<td>290</td>
<td>1.43 ± 0.79</td>
</tr>
<tr>
<td>round 4</td>
<td>92 × 98</td>
<td>1050</td>
<td>4.07 ± 2.00</td>
<td>476</td>
<td>1.46 ± 0.83</td>
</tr>
</tbody>
</table>

TABLE I: Scene details.
Fig. 9: Episodes with different Topological Complexity (TC). Each row depicts three episodes yielding distinct braids in the same scene. At the bottom right of each figure, the braid formed by the data through a $x$-$t$ side projection of the episode is plotted. The episodes on each row are organized from left to the right in order of increasing TC. In all scenes, the agents are following the right-hand traffic convention.
REFERENCES


